17. Assume that $\alpha\uparrow$ on $[a,b]$. Prove that the following statements are equivalent.

(a) $f \in R(\alpha)$ on $[a,b]$

(b) f satisfies Riemann's conditions with respect to $\alpha$ on $[a,b]$

(c) $I(f,\alpha) = \bar{I}(f,\alpha)$

18. Assume that $\alpha$ is of bounded variation on $[a,b]$. Let $v(x)$ denote the total variation of $\alpha$ on $[a,x]$ if $a < x < b$ and let $v(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, prove that $f \in R(v)$ on $[a,b]$.

19. Assume that $\sum_{n=0}^{\infty} a_n$ converges absolutely and has sum $A$ and suppose $\sum_{n=0}^{\infty} b_n$ converges with sum $B$. Prove that the Cauchy product of these two series converges and has sum $AB$.

20. State and prove Cauchy condition for uniform convergence of sequence of functions.



**NOVEMBER/DECEMBER 2024**

# GMA12 — REAL ANALYSIS - I

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Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If f is bounded on $[a,b]$, prove that f is of bounded variation on $[a,b]$.
2. State additive property of total variation.
3. Define a refinement of a partition.
4. Define we say that f satisfies Riemann's condition with respect to $\alpha$ on $[a,b]$
5. State Bonnet's theorem.
6. Write down the second mean-value theorem for Riemann-Stielties integrals.
7. Give an example for a series which is converges but it does not converge absolutely.
8. What is the meaning of big oh and little oh?
9. Define uniformly converge of a sequence.
10. Define mean convergence.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If f is monotonic on $[a,b]$ prove that the set of discontinuities of f is countable.

Or

(b) Let f be defined on $[a,b]$. Prove that f is of bounded variation on $[a,b]$ if and only if, f can be expressed as the difference of two increasing functions.

12. (a) If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on $[a,b]$, prove that $c_1 f + c_2 g \in R(\alpha)$ on $[a,b]$ for any two constants $c_1$ and $c_2$ and

$$\int_a^b (c_1 f + c_2 g)\,d\alpha = c_1 \int_a^b f\,d\alpha + c_2 \int_a^b g\,d\alpha .$$

Or

(b) Assume that $\alpha \nearrow$ on $[a,b]$, prove that if $p'$ is finer than p, $U(p',f,\alpha) \le U(p,f,\alpha)$ and $L(p',f,\alpha) \ge L(p,f,\alpha)$.

13. (a) Let $\alpha$ be of bounded variation $[a,b]$ and assume that $f \in R(\alpha)$ and $[a,b]$. Prove that $f \in R(\alpha)$ on every subinterval $[c,d]$ of $[a,b]$.

Or

(b) State and prove first mean-value theorem for Reimann-Stieljes integrals.



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14. (a) State and prove Dirichlet's test.

Or

(b) If a series is convergent with sum $S$, prove that it is also $(C,I)$ summable with Cesaro Sum $S$.

15. (a) Assume that $f_n \to f$ uniformly on S. If each $f_n$ is continuous at a point $c$ of $S$, prove that the limit function $f$ is also continuous at $c$.

Or

(b) Let f be a double sequence and let $Z^+$ denote the set of positive integers. For each $n = 1,2,\ldots\infty$ function $g_n$ on $Z^+$ is defined as $g_n(m) = f(m,n)$ if $m \in Z^+$. Assume that $g_n \to g$ uniformly on $Z^+$, where $g(m) = \lim_{n\to\infty} f(m,n)$. If the iterated limit exist, prove that the double limit $\lim_{m,n\to\infty} f(m,n)$ also exist and has the same value.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows: $V(x) = V_f(a,x)$ if $a < x \le b, v(a) = 0$. Prove that

(a) V is an increasing function of $[a,b]$

(b) $V - f$ is an increasing function on $[a,b]$.